2020

PHYSICS — HONOURS

Paper : CC-2

(Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions of the following : 2×5

(a) Show that mutually interacting forces on a system of particles have no effect on its total linear momentum.

(b) A solid sphere and a solid cylinder having same mass and same radii roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.

(c) 'In streamline flow of a Newtonian fluid two streamlines never intersect'— Explain.

(d) Prove that the areal velocity of a particle moving under a central force field is constant.

(e) What is the rotational period of a binary star consisting of two equal masses, M and separated by distance L?

(f) Find the degrees of freedom of a system of two point masses joined by a massless rigid rod in a 3-dimensional space.

2. (a) A particle is moving in a plane in such a way that its polar co-ordinates are given by $r = 2t + 3$ and $\theta = 3t - t^2$. Obtain the radial and transverse components of instantaneous acceleration.

(b) A particle of mass 'm' at rest at $(a, 0, 0)$ subjected to a force $\vec{F} = -\frac{k}{x^3}\hat{x}$, where k is a positive constant. Find the time taken by the particle to reach the origin.

(c) Given $\vec{F} = -r\hat{r}$ is a conservative force field. Find the corresponding scalar potential. 4+4+2

3. A particle of mass m moves along a trajectory given by $x = x_0 \cos \omega_1 t$, $y = y_0 \sin \omega_2 t$, where x_0 and y_0 are constants.

(a) Find the x and y components of the force. What is the condition under which the force is a central one?

(b) Find the potential energy as a function of x and y.

(c) Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved. (2+1)+3+(2+2)

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4. (a) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.

(b) Prove that total energy of a particle of mass '*m*' acted upon by a central force is given by,

$$E = \frac{L^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where L is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$, r and θ being the polar co-ordinates. 5+5

5. (a) Show how a fictitious force arises in a non-inertial frame which is moving with a constant acceleration in a given direction with respect to a fixed frame.

(b) Let S' be a reference frame which is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$,

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refer to time derivatives with respect to S and S' frames, respectively.

(c) Two reference frames, one is fixed and other one is rotating, have common origin. Obtain the equation of motion of a particle of mass '*m*' with respect to the rotating frame. Discuss about the different fictitious forces arise in the rotating frame. 2+4+4

6. (a) Show that the angular momentum vector $\vec{L}$ is not always along the same direction as the instantaneous axis of rotation.

(b) Determine the moment of inertia tensor for the configuration in which four point masses of 1, 2, 3 and 4 units are located at (1, 0, 0), (1, 1, 0), (1, 1, 1) and (1, 1, –1) units, respectively.

(c) A rigid body is rotating under the influence of an external torque $\vec{N}^{(e)}$. If the angular velocity is $\vec{\omega}$ and kinetic energy is T, show that

$$\frac{dT}{dt} = \vec{N}^{(e)} . \vec{\omega}$$

when the axes of the body co-ordinates are taken as principal axes.

(d) Indicate the principal axes for a homogeneous sphere and a cylinder in neatly labelled sketches. 2+3+3+2

7. (a) Set up Euler's equation for an incompressible fluid and establish Bernoulli's equation of fluid motion stating the assumptions used. 6

(b) A pipe of varying diameter is used to lift water by 7m. The area of cross-section of the pipe at the base is 125 cm^2 and the pressure here is $2{\cdot}5\times10^5$ Pa. The area of cross-section of the pipe at the top is 25 cm^2. The rate of flow of water is 3×10^{-2} m^3/sec. Calculate the pressure of water at the top, neglecting energy losses. 4

Or,

A copper wire of diameter 1*mm*. and length 3*meters* has Young's modulus 12.5×10^{11} dynes per sq.cm., If a weight of 10*kg.* is attached to one end, what extension is produced? If the Poisson's ratio is 0.26, what lateral compression is produced? 4